PLAN OF THE TALK

(Talk 2, Day 3)

① The Theorem
(BGG-correspondence)

② The Proposition
(Prop => Thm)

③ And the proof

(1) The Theorem

Fix r a non-negative integer and $\mathbb{K}$ a field

Def:
We denote by Λ the following DG-algebra

$\mathbb{K}\langle \xi_1, \dots, \xi_r \rangle$ The graded extend algebra generated by ξ_i s.t

$|\xi_i| < 0$ odd

$d^{\Lambda} = 0.$

$\xi_i \wedge \xi_j = (-1) \xi_j \wedge \xi_i$ (and $\xi_i \wedge \xi_i = 0$ in char $\mathbb{K} = 2$)

It has a unit
$$\eta_\Lambda : \mathbb{K} \longrightarrow \Lambda$$
and an augmentation
$$\varepsilon_\Lambda : \Lambda \longrightarrow \mathbb{K}$$

Λ has finite dimension

We denote by S the DG-algebra

$\mathbb{K}[x_1, \dots, x_r]$ the graded polynomial algebra generated by x_i s.t

$|x_i| = -|\xi_i| + 1$

$x_1 \cdot x_2 = x_2 \cdot x_1$

$d^S = 0.$

It has a unit

$$\eta_S : \mathbb{K} \to S$$

and an augmentation

$$\varepsilon_S : S \to \mathbb{K}$$

S has finite dimension level wise

THEOREM

There exists an exact functor h

$$h : D(\Lambda) \longrightarrow D(S)$$

with the property that its restriction to $D^{f}(\Lambda)$ is an equivalence.

$$h : D^{f}(\Lambda) \xrightarrow{\sim} D^{f}(S)$$

Construction of h

Def:
Consider the DG-algebra $(\Lambda \otimes_{\mathbb{K}} S, d^{\Lambda\otimes_{\mathbb{K}} S} = 0)$

It has a special element

$$\delta = \sum_{i=1}^{r} \xi_i \otimes x_i \in (\Lambda \otimes_{\mathbb{K}} S)^{+1}$$

$$\delta^2 = 0$$

Lemma

Let (E, d^E) be a DG-$(\Lambda \otimes_{\mathbb{K}} S)$-module

$$\partial : E \longrightarrow E$$

$$\partial(e) := d^E(e) + \delta . e$$

(E, ∂) is a (new) DG-$(\Lambda \otimes_{\mathbb{K}} S)$-module.

Lemma

For any N DG-S-module $N^* := \mathrm{Hom}(N, \mathbb{K})$ is a DG-S-module.

Ex.

$S^* := \mathrm{Hom}_{\mathbb{K}}(S, \mathbb{K})$ is a DG-S-module

It has two [redacted] morphisms of DG-S-mod

$$\eta_{S^*} := (\varepsilon_S)^* : \mathbb{K} \longrightarrow S^*$$

and

$$\varepsilon_{S^*} := (\eta_S)^* : S^* \longrightarrow \mathbb{K}$$

$$(\varepsilon_{S^*} \circ \eta_{S^*}) = (\eta_S^* \circ \varepsilon_S^*) = (\varepsilon_S \eta_S)^* = Id_{\mathbb{K}}$$

$$d^{S^*} = 0$$

Def

Consider the DG-$\Lambda \otimes_{\mathbb{K}} S$-module defined by

$$\left(\Lambda \otimes_{\mathbb{K}} S^*, \; d^{\Lambda \otimes_{\mathbb{K}} S^*} = 0\right)$$

we define F to be the DG-$\Lambda \otimes_{\mathbb{K}} S$-module

$$\left(\Lambda \otimes_{\mathbb{K}} S^*, \; \delta\right)$$

It has two maps

$$\eta_F : \mathbb{K} \xrightarrow{\cong} \mathbb{K} \otimes \mathbb{K} \xrightarrow{\eta_\Lambda \otimes \eta_{S^*}} \Lambda \otimes S^*$$

$$\varepsilon_F : \Lambda \otimes S^* \xrightarrow{\varepsilon_\Lambda \otimes \varepsilon_{S^*}} \mathbb{K} \otimes \mathbb{K} \cong \mathbb{K}$$

They are DG-Λ-modules morphisms

Let N be a DG-Λ-module then

$\mathrm{Hom}_\Lambda(F,N)$ is a DG-S-module

<u>Def</u>

The functor $h : D(\Lambda) \to D(S)$ is defined to be

$$h(N) := R\mathrm{Hom}_\Lambda(F,N)$$

② The Proposition

Lemma

There exists an isomorphism of DG-S-modules

$$\mathrm{Hom}_\Lambda(F; \mathbb{K}) \cong S$$

Proposition

The following statements hold

1) $\varepsilon_F : F \longrightarrow \mathbb{K}$ is a q. iso of DG-Λ-module
2) $\mathrm{Hom}_\Lambda(F, -)$ preserves the q. iso
3) The map $\overline{\sigma}(S) : S \longrightarrow \mathrm{Hom}_\Lambda(F, F)$ is a q. iso of DG-S-module

Prop $\Rightarrow$ Thm

$$\lambda(\mathbb{K}) : F \overset{L}{\underset{S}{\otimes}} \mathrm{Hom}_\Lambda(F, \mathbb{K}) \cong F \overset{L}{\underset{S}{\otimes}} S \cong F \underset{S}{\otimes} S$$

$$\cong F \xrightarrow{\varepsilon_F} \mathbb{K}$$

$$\sigma(S) : S \xrightarrow{\overline{\sigma}(S)} \mathrm{Hom}_\Lambda(F, F \overset{L}{\underset{S}{\otimes}} S) \cong \mathrm{Hom}_\Lambda(F, F)$$

3) The proof

1)

$$id_{\mathbb{K}} = \mathbb{K} \xrightarrow{\eta_F} F \xrightarrow{\varepsilon_F} \mathbb{K}$$

$$\mathbb{K} \xleftarrow{\eta_F^*} F^* \xleftarrow{\varepsilon_F^*} \mathbb{K}$$

and

$$F^* \cong (\Lambda^* \otimes_{\mathbb{K}} S, \delta^*)$$

bigrading ξ_i and x_i with homological degree $+1$ and 0 one obtain the graded Koszul complex on $x_1, \dots, x_c \in S$.

Then η_F^* is a q. iso.

$id_{\mathbb{K}} = \eta_F^* \circ \varepsilon_F^*$

then ε_F^* is a q. iso

(actually: 10/4 ...) 11/4

3)

$$\mathrm{id}(S) \doteq S \xrightarrow{\bar{\sigma}(S)} \mathrm{Hom}_\Lambda(F,F) \xrightarrow{\mathrm{Hom}_\Lambda(F,\varepsilon)} \mathrm{Hom}_\Lambda(F,\mathbb{K}) \cong S$$

1) ε q.iso
2) $\mathrm{Hom}(F,\varepsilon)$ q. iso

$F \overset{L}{\underset{S}{\otimes}} -$ ①

$\parallel$

$$
\begin{tikzcd}
D(\Lambda) \arrow[r, "\Delta"'] & D(S) \arrow[l, bend right, "t"'] \\
\mathrm{Thick}_\Lambda(\mathbb{K}) \arrow[u, hook] \arrow[r] \arrow[d, phantom, "\cong"] & \mathrm{Thick}_S(\ ) \arrow[l] \arrow[u, hook] \arrow[d, phantom, "\cong"] \\
D^f(\Lambda) \arrow[r] & D^f(S) \arrow[l]
\end{tikzcd}
$$

$h = RHom_\Lambda(F,-) = Hom_\Lambda(F,-)$ ① ②

$RHom(F,\mathbb{K})$ ③

$\mathrm{Thick}_S(\ ) \cong \mathrm{Thick}_S(S)$ ②

Λ coo

non-positive

$H^0\Lambda = \mathbb{K}$ semisimple

$\parallel$

∇

$h : D^f(\Lambda) \rightleftarrows D^f(S) : t$ ①

$\lambda : th \to id_{D(\Lambda)}$

$\sigma : id_{D(S)} \to ht$ ①

1 = 10:30 - 10:45 -

2 = 10:45 - 11:00 +

3 = 11:00 - 11:15

4 = 11:15 - 11:30